Modeling of the Nonlinear Signal Propagation in Multi-Mode Fibers with SDM

Georg Rademacher and Klaus Petermann
Technische Universität Berlin, FB Hochfrequenztechnik, Einsteinufer 25, 10587 Berlin, Germany
georg.rademacher@tu-berlin.de

Abstract: We present an analytical model for the nonlinear interference in multi-mode fiber SDM systems and verify it with numerical simulations. The model allows general predictions for nonlinear system implications even in fibers with many modes.

© 2015 Optical Society of America
OCIS codes: 060.2330, 060.4230.

1. Introduction

Space-Division Multiplexing (SDM) is widely discussed as a very promising candidate to overcome the upcoming capacity crunch in long-haul fiber optical transmission systems [1]. For a drastic increase of the per-fiber capacity of a hundredfold or larger, while keeping the cladding diameter within current fiber measures, multi-mode fibers or at least multi-core-multi-mode fibers are required. As in current single-mode systems, the maximum transmission rate in SDM systems is fundamentally limited by Kerr-induced nonlinear effects. Since the mode-fields of a multi-mode fiber share the core’s cross-section, nonlinear signal interactions cannot only occur within, but also between fiber modes. In order to assess the full performance potential of multi-mode fibers and therefore their ability to serve as the optical fiber for the future, it is inevitable to establish a detailed understanding of the nonlinear transmission effects. Effects like intermodal cross-phase modulation and four-wave mixing have been investigated, both theoretical [2–6] and experimental [7], identifying the differential mode delay (DMD) and linear coupling as crucial parameters for intermodal nonlinear interactions. With this paper, we want to establish a more general understanding of the strength and the impact of intra- and intermodal nonlinear interactions. We thus extend our analytical modeling of the nonlinear interaction in multi-mode fibers [4] that only considered one polarization to a more complete scenario that includes two strongly coupled polarizations per mode and validate this model by numerical simulations.

2. Analytical Model for the Nonlinear Interference in Multi-Mode Fibers

Splitt et al. presented an analytical approach to estimate the overall nonlinear signal distortion in single-mode fibers [8] that was later widely discussed and enhanced to cover more transmission scenarios. Carena et al. presented a very detailed derivation in [9]. The fundamental idea behind this model is that a WDM comb can be constructed by many spectral lines that all co-propagate inside the fiber and interact through FWM and sum up to an additional Gaussian noise-like term $G_{NL}$. In multi-mode fibers, the nonlinear Gaussian noise needs to incorporate FWM products from intermodal nonlinear interactions. It needs to be evaluated for every mode $p$ separately as $G_{NL}^{(p)}$ that can then be used to estimate the mode’s receiver OSNR [9] as:

$$OSNR(p) = \frac{P_{ch}}{B_{ref}(G_{NL}^{(p)} + N_{lin})} \quad \text{with} \quad G_{NL} = N_{sp} \cdot G_{NL,ss}$$

(1)

Where $P_{ch}$ is the signal power per channel, $N_{lin}$ is the total linear noise contribution, $B_{ref}$ is the reference bandwidth of typically 12.5 GHz and $G_{NL,ss}$ is the single-span contribution of the nonlinear noise density that we consider to increase linearly with the number of transmission spans $N_{sp}$ [9]. For the multi-mode scenario, we split $G_{NL,ss}$ into an intra- and an intermodal term: $G_{NL,ss} = G_{NL,intra}^{(p)} + G_{NL,inter}^{(p)}$. While generally frequency dependent, we evaluate $G_{NL,ss}^{(p)}$ at the center of the C-Band, $f_0$, and consider it to be constant over the bandwidth of the channel under test as [9]:


$G_{\text{NL, intra}}^{(p)} = \left( \frac{8}{9} \gamma_{pp} \right)^2 \frac{B_{\text{opt}}}{-B_{\text{opt}}/2} \int_{-B_{\text{opt}}/2}^{B_{\text{opt}}/2} G_{\text{TX}}(\Delta f_1)G_{\text{TX}}(\Delta f_2)G_{\text{TX}}(\Delta f_1 + \Delta f_2) \left| \eta(\Delta \beta_{\text{intra}}^{(pp)}) \right|^2 d\Delta f_1 d\Delta f_2$

$G_{\text{NL, inter}}^{(p)} = 2 \sum_{q \neq p} \left( \frac{2}{3} \gamma_{pq} \right)^2 \frac{B_{\text{opt}}}{-B_{\text{opt}}/2} \int_{-B_{\text{opt}}/2}^{B_{\text{opt}}/2} G_{\text{TX}}(\Delta f_1)G_{\text{TX}}(\Delta f_2)G_{\text{TX}}(\Delta f_1 + \Delta f_2) \left| \eta(\Delta \beta_{\text{inter}}^{(pq)}) \right|^2 d\Delta f_1 d\Delta f_2$  \hspace{1cm} (2)

A more detailed derivation of eq. (2) can be found in [10]. While in [4], a triple summation was required to include all intermodal nonlinear interactions, eq. (2) shows that a single summation over all fiber modes is sufficient. The first row in eq. (2) represents the effect of intramodal nonlinear interaction; the second row includes effects that originate from the interaction between the considered mode $p$ and all other modes. $\gamma_{pq}$ is the intra- / intermodal nonlinear coefficient [4]. The factors of $8/9$ for the intramodal term and $2/3$ for the intermodal term correspond to those that were introduced in the propagation equation [2,3] due to the strongly coupled polarizations. By adapting these factors, it is possible to include other linear coupling scenarios [2]. The factor of 2 in front of the sum represents the fact that two different intermodal FWM processes are present that are independent of another [7] but with equal impact. $G_{\text{TX}}$ is a signals’ exemplary power spectral density, as shown in Fig. 1 (a). $\eta(\Delta \beta)$ is the FWM efficiency, defined as:

$$\eta(\Delta \beta_{\text{pq}}^{(pq)}) = \frac{1 - e^{-2\alpha L}e^{-j\Delta \beta_{\text{pq}}^{(pq)}(\Delta f_1, \Delta f_2)L}}{2\alpha + j\Delta \beta_{\text{pq}}^{(pq)}(\Delta f_1, \Delta f_2)} \left| \Delta \beta_{\text{pq}}^{(pq)}(\Delta f_1, \Delta f_2) = \left( \beta_{1}^{(p)} - \beta_{1}^{(q)} \right) \right|^2 \Delta f_2 - \beta_{2}^{(p)} \Delta f_2 4\pi^2 \Delta f_1 \Delta f_2$$  \hspace{1cm} (3)

with $L$ being the length of one fiber span and $\alpha$ the attenuation coefficient. At the fixed center frequency $f_0$, where we want to evaluate the nonlinear Gaussian noise density, the phase matching condition $\Delta \beta$ in eq. (3) is only a function of the two frequency differences $\Delta f_1$ and $\Delta f_2$, due to the frequency condition of the FWM process [4]. The spectral arrangement when fulfilling this condition is also shown in Fig. 1 (a). For the significant FWM processes [10], where two spectral components travel in one mode and two in the other, $\Delta \beta_{\text{pq}}^{(pq)}$ can be seen in eq. (3) with $\beta_{1}^{(p)}$ being the group-delay parameter and $\beta_{2}^{(p)}$ the chromatic dispersion parameter of the $p$’s fiber mode. Fig. 1 shows the FWM efficiency for (b) an intra- and (c) an intermodal FWM process. The marked square area shows the (simplified) integration boundaries from eq. (3). While the maximum of the FWM efficiency is in the center of the plot for the intramodal case, as shown in fig. 1 (b), it moves for the intermodal case. The shift depends mainly on the differential mode delay (DMD) between the interacting modes [4]. If the DMD is zero, the FWM efficiency is equal for the intra- and intermodal FWM process, as it can be seen from eq. (3). If the DMD increases, the integration over the marked region leads to a smaller and ultimately negligible contribution, indicating that the intermodal nonlinear interaction vanishes.

Fig. 1: (a) Power spectral density of a WDM signal and representation of the frequency condition for FWM processes [4]. Contour plot of the FWM efficiency $\eta(\Delta \beta)$ for an (b) intra- and (c) intermodal nonlinear FWM process.

### 3. Verification and Discussion

As the single-mode nonlinear Gaussian noise model has been widely proven [9], we are here mainly interested in comparing the relative strength of the intra- and the intermodal nonlinear signal distortion. Thus, we introduce the ratio $\rho = G_{\text{NL, intra}}^{(p)} / G_{\text{NL}}^{(p)}$. When $\rho$ approaches one, the intermodal nonlinear interaction vanishes. For the numerical verification, we perform a split-step Fourier simulation to solve the multi-mode nonlinear Schroedinger equation, as reported e.g. in [3, 6], with 9 Nyquist-spaced 28 GBaud QPSK signals at -8 dBm in each of three spatial modes.
We assess the nonlinear noise as the variance of the center channel's constellation points, simulating the cases where both, intra- and intermodal nonlinear interactions and when only intramodal interaction is present ($\gamma_{pq} = 0$). For a linear increase of the nonlinear Gaussian noise with transmission distance, even with only 9 WDM channels, we simulate a pump-probe scenario. The center channel of the LP$_{01}$ and the channels in the LP$_{11}$ modes with strongest intermodal interaction are launched at a reduced power of -15 dBm.

Fig. 2 shows the evolution of $\rho$ for the LP$_{01}$ mode as a function of the transmission distance at zero DMD. After about 600 km, the numerical results approach the analytically predicted value. As the value of $\rho$ approaches 0.64, it can be concluded that the overall nonlinear signal distortion that originates from signals in different fiber modes is less strong than the signal distortion coming from signals in the mode itself. A small deviations between simulations and analytic solutions can be observed that indicates a minor underestimation of the strength of intermodal nonlinear interactions. Fig. 2 (b) shows $\rho$, for the LP$_{01}$ mode, as a function of the DMD at a transmission distance of 2000 km. $\rho$ approaches one for DMD values above 25 ps/km, meaning that the intermodal nonlinear interaction vanishes. Long-haul, high capacity transmission systems that make use of e.g. the entire C- Band (about 4 THz bandwidth), behave similar to the scenario with little DMD, as the combination of the large bandwidth and low DMD that is required for efficient MIMO processing, inevitably leads to a strong intermodal nonlinear interaction.

4. Conclusion

We have presented and validated an easy-to-use analytical model to assess the nonlinear signal distortion in multimode fiber-based Space-Division Multiplexed transmission systems. The analytical model allows a good prediction of the overall nonlinear system implications and is easily scalable to systems with large numbers of modes with various DMD-values and linear coupling scenarios.

Acknowledgment

This work was funded by the German Research Foundation (DFG project PE 319/29-1).

References